BAMO
Bay Area Mathematical Olympiad

12th Bay Area Mathematical Olympiad and 1st Teachers' Olympiad

February 21, 2010

Problems

1 Three horses, Abel, Bourbaki, and Cantor, are in a race. Abel has winning odds of $4:1$ (that is, if you bet on this horse, and it comes in first, you receive your bet plus four times the amount of your bet). Bourbaki has odds of $3:1$ and Cantor has odds of $1:1$. If the horse that you bet on does not come in first, you lose your bet.

Is it possible to arrange your bets so that you will make a profit, no matter which horse wins?

Present your solution in a way that would be understandable to a sophisticated algebra 1 student.

2 What positive integers can be written in the form $ab+a+b$ where a and b are positive integers?

Present your solution in a way that would be understandable to a sophisticated algebra 1 student.

3 Students often interpret problems in ways that their teacher or textbook might not have expected. An answer that might seem wrong at first may be a perfectly correct answer to a different question. For example:

> Four chairs are arranged around a circular table. How many ways can Abi, Broderick, Cayley, and DeSean sit around the table?

Interpret this problem in as many different ways as you can, and solve the problem in each case.

4 Explain in as many different ways as you can which of the following two numbers is larger:

$$\sqrt{2009}+\sqrt{2011} \qquad \text{and} \qquad 2\sqrt{2010}$$

5 Find and correct the mistakes in the following fictional section of an algebra teacher's guide. (The mistakes may be found in the statement of the problem and/or the answer and/or the solution.)

Problem: Solve the equation

$$\sqrt{x^2-1}=(x+5)\sqrt{\frac{x+1}{x-1}}$$

Answer: $x=-1$.

Solution: Rewrite the equation by factoring, as

$$\sqrt{(x+1)(x-1)}=(x+5)\sqrt{\frac{x+1}{x-1}}.$$

Then it is clear that if $x=-1$, both sides equal 0. If $x\neq -1$ then we can divide both sides by $x+1$, obtaining

$$\sqrt{x-1}=(x+5)\sqrt{\frac{1}{x-1}}.$$

Now we see that $x\neq 1$ and then this equation can be true if and only if $\left(\sqrt{x-1}\right)^2=x+5$. But of course $x-1=x+5$ has no solution, so we have only $x=-1$.

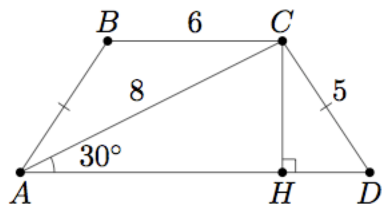
6 Find and correct the mistakes in the following fictional section of a geometry teacher's guide. (The mistakes may be found in the statement of the problem and/or the answer and/or the solution.)

Problem: In the isosceles trapezoid $ABCD$ with bases AD and BC, $AB = DC = 5$cm, $BC = 6$cm, $AC = 8$cm, and $\angle CAD = 30^\circ$. Find the area of the trapezoid.

Answer: 36cm^2.

Solution: Drop altitude CH as in the diagram. Since $\angle CAD = 30^\circ$, $CH = AC/2 = 4$cm. Then triangle CDH is a $3-4-5$ right triangle, so $DH = 3$cm. Since the trapezoid is isosceles, $AD = BC + 2DH = 12$cm. Hence the area of the trapezoid is equal to

$$\left(\frac{AD+BC}{2}\right) CH = 36\text{cm}^2.$$



7 Using only the fractions $\frac{1}{3}$ and $\frac{2}{5}$, write a sequence of word problems accessible to sixth graders that have as many different answers as you can think of. Problems that are as similar as possible (using the same theme or context or having some other strong relationship) are better than problems that are unrelated.

Answers may include $\frac{1}{15}$, $\frac{2}{15}$, $\frac{6}{5}$ or $\frac{5}{6}$ (but probably not both unless you have a very good reason to), $\frac{11}{15}$, and I hope at least one or two more of your own invention.

Give the answer to each problem, but you do not need to show any steps of the solution.

You may keep this exam. **Please remember your ID number!** Our grading records will use it instead of your name.

You are cordially invited to attend the **BAMO 2010 Awards Ceremony**, which will be held at the Mathematical Sciences Research Institute, from 11–2 on Sunday, March 7. This event will include lunch, a mathematical talk by Thomas Banchoff of Brown University, and the awarding of dozens of prizes. Solutions to the problems above will also be available at this event. Please check with your proctor for a more detailed schedule, plus directions.