13th Bay Area Mathematical Olympiad and 2nd Teachers' Olympiad

March 5, 2011

Problems

1 Two teachers are investing in the same mutual fund which changed in price every day. Over the past month, the average price of a share was \$20. Each day of that month, the first teacher bought one share while the second teacher bought \$20 worth of the fund. Which teacher spent more money, and which one owns more shares?

(For the purposes of this problem, ignore that mutual funds can ordinarily only be bought in quantities rounded to the nearest thousandth or ten-thousandth of a share.)

2 Determine the sum of the 100 numbers that appear in a 10×10 multiplication table in as many different ways as you can.

3 What must be true about a and b if $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$ both have two (not necessarily distinct) integer solutions?

4 A student was recently solving this problem:

> Two red and three white beads are arranged in a circular necklace. How many different arrangements are possible?

The student's solution read as follows:

> First of all, let's put the beads in order in a line. There are $\binom{5}{2} = 10$ ways to do this.
>
> Next, we'll put them in a circle. The circular necklace could be rotated any of five different ways, so there are $\frac{10}{5} = 2$ ways to make a circular necklace.
>
> Finally, each necklace could also be mirror imaged by flipping it over. Thus there is only $\frac{2}{2} = 1$ necklace.

How would you go about convincing the student that this answer is incorrect? How would you lead up to this problem with a sequence beginning with trivial problems and working up to this one? What is the correct answer? Can you solve similar problems, say with 14 red and 6 white beads?

5 A student has recently learned a lot of shortcuts for finding remainders. For instance, they know that to find the remainder when $2011 \cdot 2012$ is divided by 7, you don't need to multiply and then divide and take the remainder; instead, you can see that 2011 leaves remainder 2 and 2012 leaves remainder 3, and so $2011 \cdot 2012$ leaves remainder $2 \cdot 3 = 6$.

This same student claims that you can find the remainder when 2011^{2012} is divided by 7 using the same process: take the remainders, and $2^3 = 8$ leaves remainder 1. Is this answer justified? Why or why not?

Turn over the page to find the remaining problems!

6 A student might see the analogy "exponentiation is to multiplication as multiplication is to addition". Discuss some ways in which this analogy could be useful to students and illustrate some ways in which this analogy could mislead students. Give specific examples.

7 In giving a test on the distributive property, it occurs to you that it would be convenient to have two forms of the test with different questions but the same answers. Then you could quickly check the exams by looking at the answers, and if you had reason to suspect cheating, you could look more closely at the students' work to see if the work corresponds to their particular problem.

Is this possible? Can there be two different distributive property questions with the same answer? If yes, give an example. If no, explain why not.

You may keep this exam. **Please remember your ID number!** Our grading records will use it instead of your name.

You are cordially invited to attend the **BAMO 2011 Awards Ceremony**, which will be held at the Mathematical Sciences Research Institute, from 11–2 on Sunday, March 13. This event will include lunch, a mathematical talk by Professor Bernd Sturmfels, and the awarding of dozens of prizes. Solutions to the problems above will also be available at this event. Please check with your proctor for a more detailed schedule, plus directions.